Public Key Cryptography and Protocols

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Cryptographic Ciphers: Recap

A cryptographic cipher is a pair of Encrypt and Decrypt algorithms such that given *plaintext* P, encryption key K_1 and decryption key K_2 then

$$D(K_2, E(K_1, P)) = P$$

- ☐ In absence of knowledge about K_2, it must be not be feasible to recover P from the ciphertext $E(K_1, P)$.
- ☐ Given P and $E(K_1, P)$, it must not be feasible to recover K_1

Note that the *plaintext* P can be any data, including plain text.

We call $E(K_1, P)$ the *ciphertext*.

Symmetric Cryptography: $K1 = K2$. This may also be called *secret key cryptography*.

Asymmetric Cryptography: $K1 \neq K2$. This is commonly called *public-key cryptography*.

Modular Arithmetic: Exponentiation

- □ Calculating $2^{16} = 2 \times 2 \times \cdots \times 2$ requires 15 "$\times$" operations to raise 2 to the power of this 4-bit number. In general computing g^e in this way is bounded by e operations, i.e., is exponential in the *size* of e.
- □ Improve strategy by calculating 2^{16} in 4 operations:

$$2^{16} = (2^8)^2 = ((2^4)^2)^2 = (((2^2)^2)^2)^2$$

In general, computing g^e in this way needs at best $\mathcal{O}(log_2(e))$ operations and is linear in the *size* of e.

- □ Generalize strategy: (observe $x^{2n} = (x^n)^2$ and $x^{n+1} = x \times x^n$)

$$\begin{aligned} 2^9 &= 2 \times 2^8 \\ &= 2 \times ((2^2)^2)^2 \end{aligned}$$

$$\begin{aligned} 2^7 &= 2 \times 2^6 \\ &= 2 \times (2^3)^2 = 2 \times (2 \times 2^2)^2 \end{aligned}$$

Modular Arithmetic: Exponentiation

Calculate

$$5^{58} \bmod 97 = (\text{a really big 40 digit number}) \bmod 97 = 44$$

There's an easy way to manage such large numbers by using property:

$$(a \times b) \bmod c = ((a \bmod c) \times (b \bmod c)) \bmod c$$

Example

$$\begin{aligned} 5^{58} \bmod 97 &= (5^{29} \bmod 97)^2 \bmod 97 \\ &= ((5 \bmod 97) \times (5^{14} \bmod 97)^2 \bmod 97)^2 \bmod 97 \\ &= \cdots \\ &= (5 \times 48^2 \bmod 97)^2 \bmod 97 \\ &= 44 \bmod 97 \end{aligned}$$

Modular Arithmetic: Exponentiation Algorithm

From the observations on the previous two slides we can conclude that it is computationally *feasible* to compute $g^x \bmod n$ for large x, n.

Algorithm fastExp(g,e,n) implements $\mathsf{g}^{\mathsf{e}} \bmod \mathsf{n}$:

```
fastExp(g,0,n)   = 1

fastExp(g,e,n)   = if odd(e) then
                      return (g * fastExp(g, e-1, n) mod n)
                    else
                      return (sqr(fastExp(g, e/2, n) mod n))
```

Modular Arithmetic: Discrete Logarithm

- Let g be a *primitive root* of n, i.e., the powers (modulo n) of g generate all integers from 1 to $n-1$: $g^1 \bmod n, g^2 \bmod n, \ldots, g^{n-1} \bmod n$ are distinct, consisting of integers 1 thru $n-1$ in some permutation.
- For any integer x, $0 < x < n$ and primitive root g of n one can find a unique exponent e such that the equation $x = g^e \bmod n$ holds.
- e is the *"discrete logarithm"* of x (generator g, modulus n).
- For example, the discrete log of 44 (generator 5, modulus 97) is 58.
- Given g, n then computing e from $x = g^e \bmod n$ is hard since, we intuitively have to 'test' the equation for every $e = 1, 2, 3, \ldots n-1$ (for large n).
- Computing the discrete logarithm is exponential in the size of n and is not feasible if n is large.

Diffie Hellman Key Exchange[1976]

Alice and Bob agree on an encryption key over a network. Steps:

- ☐ Alice and Bob agree on a good g and n. Can be done in public.
- ☐ Alice picks a large random integer x and computes $X = g^x \bmod n$. She keeps x secret, but it doesn't matter who knows X (discrete log).
- ☐ Bob behaves in the same way, picking y and computing $Y = g^y \bmod n$.
- ☐ Alice sends X to Bob, and Bob sends Y to Alice.
- ☐ Alice computes $k = Y^x \bmod n$ and Bob computes $k' = X^y \bmod n$.
- ☐ By modular arithmetic $k = g^{x*y} \bmod n = k'$. k is secret key between Alice and Bob. No one listening on the channel can compute key g^{x*y} in a reasonable amount of time (calculate discrete log of X and Y).

Diffie Hellman Key Exchange: Bucket Brigade Attack

Basic DH Protocol (in any order):

$$\begin{array}{lll} \text{Msg1} & A \to B: & g^x \text{ mod } n \\ \text{Msg2} & B \to A: & g^y \text{ mod } n \end{array}$$

Neither party really knows with whom it shares secret $k = g^{xy} \bmod n$.

Eve routes messages between Alice and Bob

$$\begin{array}{lll} \text{Msg}\alpha 1 & A \to B[E]: & g^x \text{ mod } n \\ \text{Msg}\beta 1 & A[E] \to B: & g^z \text{ mod } n \\ \text{Msg}\alpha 2 & B[E] \to A: & g^z \text{ mod } n \\ \text{Msg}\beta 2 & B \to A[E]: & g^y \text{ mod } n \end{array}$$

Alice uses $k_a = g^{xz} \bmod n$ to ‘speak’ with Bob, Bob uses $k_b = g^{zy} \bmod n$ to ‘speak’ with Alice, and Eve knows both keys, while carrying out a man-in-the-middle attack.

Diffie Hellman Key Exchange

The basic DH protocol does not provide authentication.

We need to be careful how we extend the protocol to include authentication. For example, simply adding passwords as follows does not work:

Msg1	$A \rightarrow B:$	$g^x \bmod n$
Msg2	$B \rightarrow A:$	$g^y \bmod n$
Msg3	$A \rightarrow B:$	$\{Alice's\ password\}_{g^x \bmod n}$
Msg3	$B \rightarrow A:$	$\{Bob's\ password\}_{g^x \bmod n}$

In this case the attacker can still carry out the bucket brigade/man in the middle attack.

DH Exchange: achieving Authentication

Think of $X = g^x \bmod n$ as something that Alice makes public, while x is something she keeps private.

If Bob can be sure that the public $X = g^x \bmod n$ was generated/owned by Alice and Alice can be sure that the public $Y = g^y \bmod n$ was generated/owned by Bob, then we do have an authentic connection between Alice and Bob. In this case, the DH protocol can be used to establish a shared secret key.

Msg1	$A \rightarrow B:$	g^x mod n
Msg2	$B \rightarrow A:$	g^y mod n

However, publicly stating 'Alice$=X$' on the network is not sufficient.

DH is a very common protocol for exchanging keys when the authenticity (owners) of X and Y are known;

Other protocols are used to establish authenticity of X and Y.

Public Key Cryptography Overview

Given plaintext P, ciphertext C, key K and its inverse key K^{-1}:

$$C = \{P\}_K \qquad P = \{C\}_{K^{-1}}$$

Intuitively, think of K as encryption key and K^{-1} as decryption key.

In absence of knowledge about K, not feasible to recover P from C.
Not feasible to recover K from P and/or C.
Not feasible to determine K^{-1} from K.

Alice keeps K_A^{-1} secret ("private key"), publishes K_A ("public key"). Bob does same for different K_B, K_B^{-1}.

Confidentiality: Alice sends $\{M\}_{K_B}$ to Bob, trusting that Bob does not share K_B^{-1} with anyone.

Authentication: Alice *signs* message M as $\{M\}_{K_A^{-1}}$. Anyone can confirm her signature by decrypting with her public key K_A.

Combined: $Alice \rightarrow Bob : \{\{M\}_{K_A^{-1}}\}_{K_B}$. A signed letter in an envelope!

RSA Public Key Cryptography (Sketch)

Choose two large prime numbers p and q and let $n = p \times q$.

The "totient" $\phi(n)$ of n is the number of numbers less than n with no factors in common with n.

Pick integer $e < n$ relatively prime to $\phi(n)$.
Find a second integer d such that $e \times d \bmod n = 1$.
It turns out that knowing $\phi(n)$ makes it easy/feasible to find this d.

The public key is (e, n), the private key is (d, n).

Let $m < n$ represent a message, then

- ☐ $c = m^e \bmod n$ is the encryption of m with the public key, and
- ☐ $m = c^d \bmod n$ is the decryption of m with the private key.

Break an arbitrary length message into $\lceil log_2(n) \rceil$ sized-blocks and encrypt a block at a time ($\lceil log_2(n) \rceil$ gives number of bits needed to represent n).

The size of the key in RSA is typically considered to be the size of the modulus.

RSA Example

Pick $p = 47$; $q = 71$; $n = pq = 3337$.

$\phi(n) = (p-1)(q-1) = 3220$.

Randomly pick $e = 79$ such that $gcd(e, \phi(n)) = 1$

Compute $d = e^{-1}[\text{mod}\,\phi(n)] = fastexp(79, 3320-1, 3220) = 1019$.

To encrypt message 688232686, break into blocks 688 232 686.

b	$c = b^e \bmod n$	$c^d \bmod n$
688	$688^{79} \bmod 3337 = 1570$	$1570^{1019} \bmod 3337 = 688$
232	$232^{79} \bmod 3337 = 2756$	$2756^{1019} \bmod 3337 = 232$
686	$686^{79} \bmod 3337 = 2091$	$2091^{1019} \bmod 3337 = 686$

Note this example does not consider the problem of traffic analysis: an attacker can detect patterns in the ciphertext. How would we prevent this?

Note: in practice we don't use RSA to encrypt bulk data.

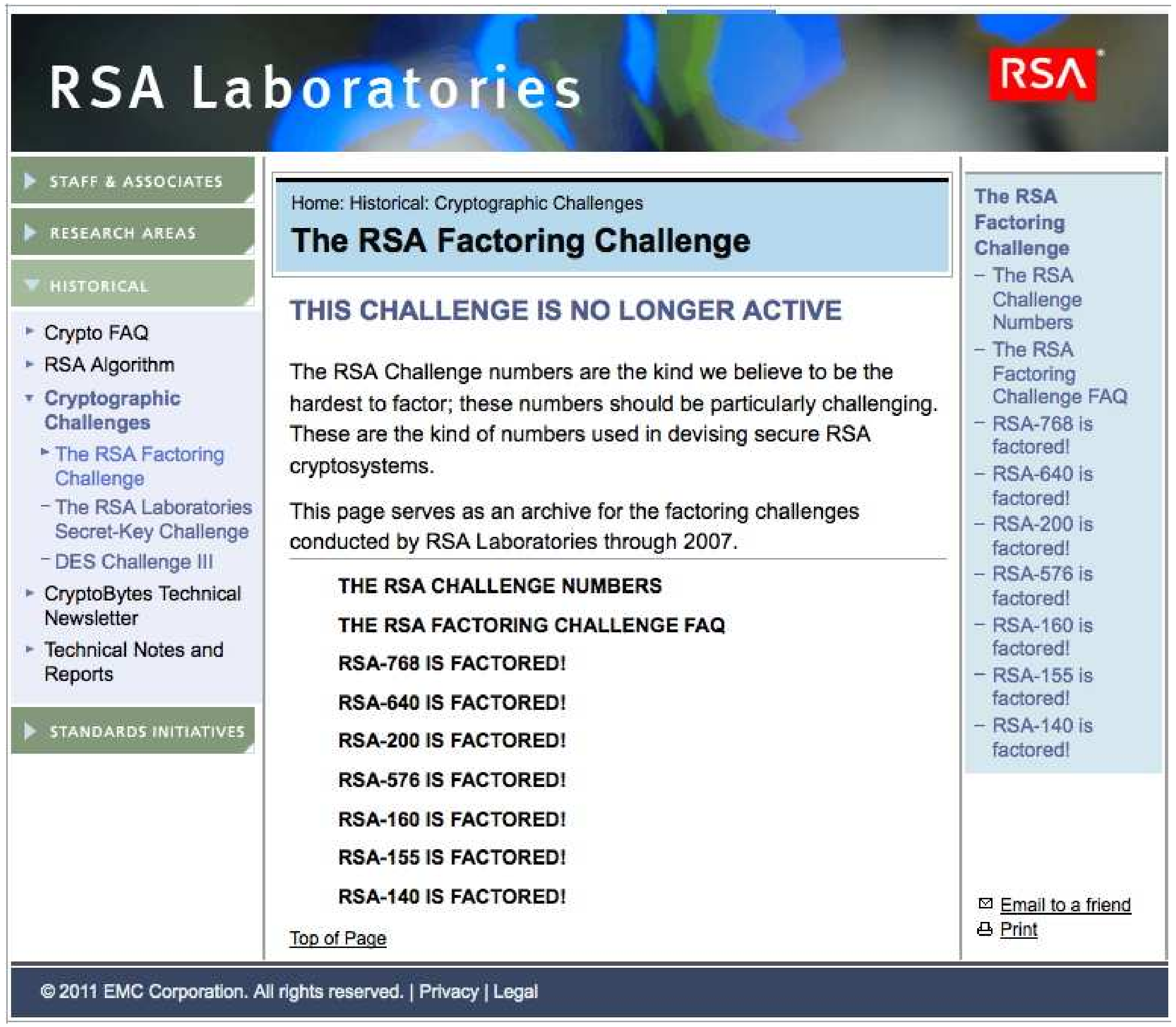
RSA Public Key Scheme

We do not consider the mathematics behind RSA. However, there are a number of important requirements that have to be observed.

- ☐ Given the public key (e, n), then the decryption key can be computed if the primes p and q are publicly known. Therefore, the primes p and q must not be revealed by the principal that generates the public/private key pair.
- ☐ The primes p and q have to be large enough to ensure that it is not possible to factor them from the public modulus n.
- ☐ In practice, a principal generates random p and q and computes (e, n) and (d, n) and then discards p and q. The principal keeps (d, n) private and makes key (e, n) public.
- ☐ In practice, primes should be at least 1024 bits long, and be ‘suitable’.
- ☐ Many other practical requirements on the properties of the numbers.

Use RSA implementations that are compliant with standards!

RSA Factoring Challenge: remember the public key is public



Home: Historical: Cryptographic Challenges

The RSA Factoring Challenge

THIS CHALLENGE IS NO LONGER ACTIVE

The RSA Challenge numbers are the kind we believe to be the hardest to factor; these numbers should be particularly challenging. These are the kind of numbers used in devising secure RSA cryptosystems.

This page serves as an archive for the factoring challenges conducted by RSA Laboratories through 2007.

THE RSA CHALLENGE NUMBERS
THE RSA FACTORING CHALLENGE FAQ
RSA-768 IS FACTORED!
RSA-640 IS FACTORED!
RSA-200 IS FACTORED!
RSA-576 IS FACTORED!
RSA-160 IS FACTORED!
RSA-155 IS FACTORED!
RSA-140 IS FACTORED!

Top of Page

The RSA Factoring Challenge
- The RSA Challenge Numbers
- The RSA Factoring Challenge FAQ
- RSA-768 is factored!
- RSA-640 is factored!
- RSA-200 is factored!
- RSA-576 is factored!
- RSA-160 is factored!
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- RSA-140 is factored!

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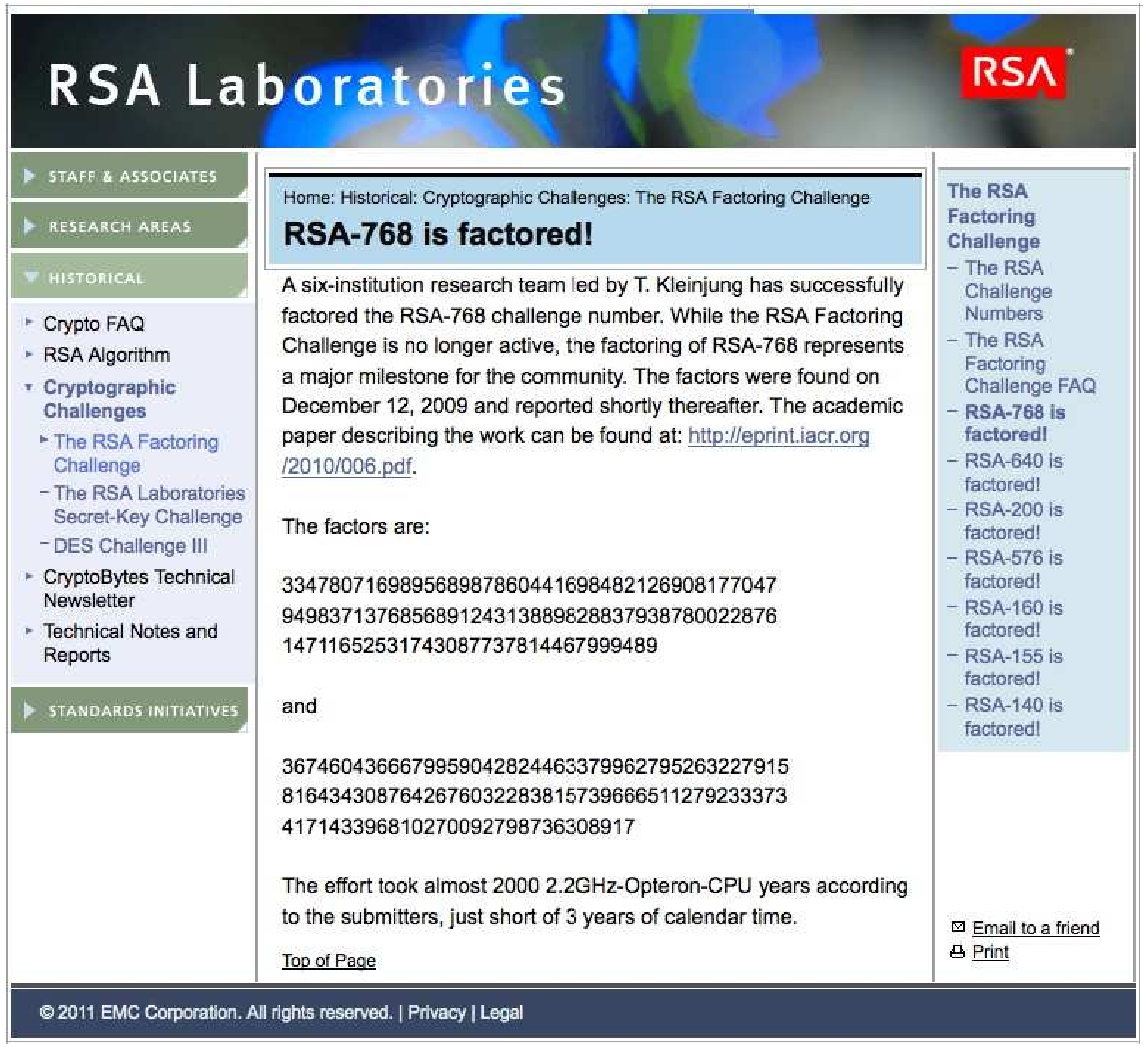
RSA Factoring Challenge



Home: Historical: Cryptographic Challenges: The RSA Factoring Challenge

RSA-768 is factored!

A six-institution research team led by T. Kleinjung has successfully factored the RSA-768 challenge number. While the RSA Factoring Challenge is no longer active, the factoring of RSA-768 represents a major milestone for the community. The factors were found on December 12, 2009 and reported shortly thereafter. The academic paper describing the work can be found at: http://eprint.iacr.org/2010/006.pdf.

The factors are:

3347807169895689878604416984821269081770479498371376856891243138898288379387800228761471165253174308773781446799949

and

3674604366679959042824463379962795263227915816434308764267603228381573966651127923337341714339681027009279873630 8917

The effort took almost 2000 2.2GHz-Opteron-CPU years according to the submitters, just short of 3 years of calendar time.

Top of Page

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Recommended Key Sizes

From NIST Special Publication 800-57, 2008, *Recommendation for Key Management - Part 3: Application-specific key management guide (draft)*.

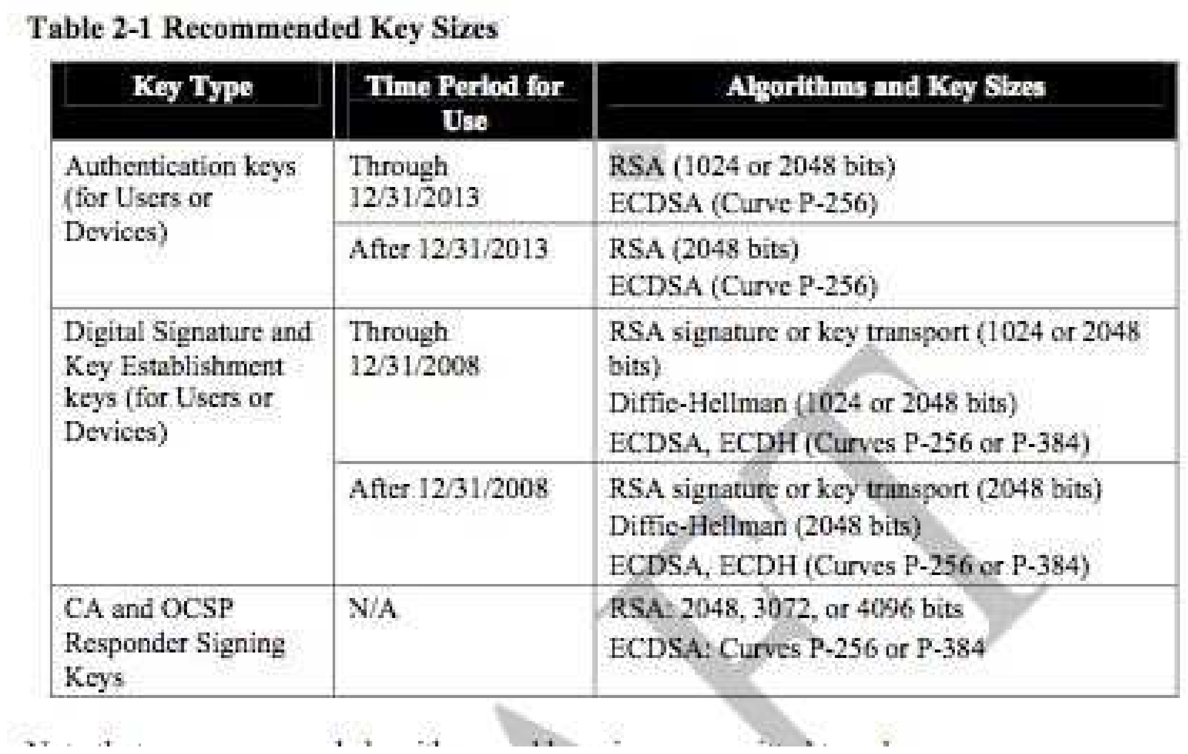
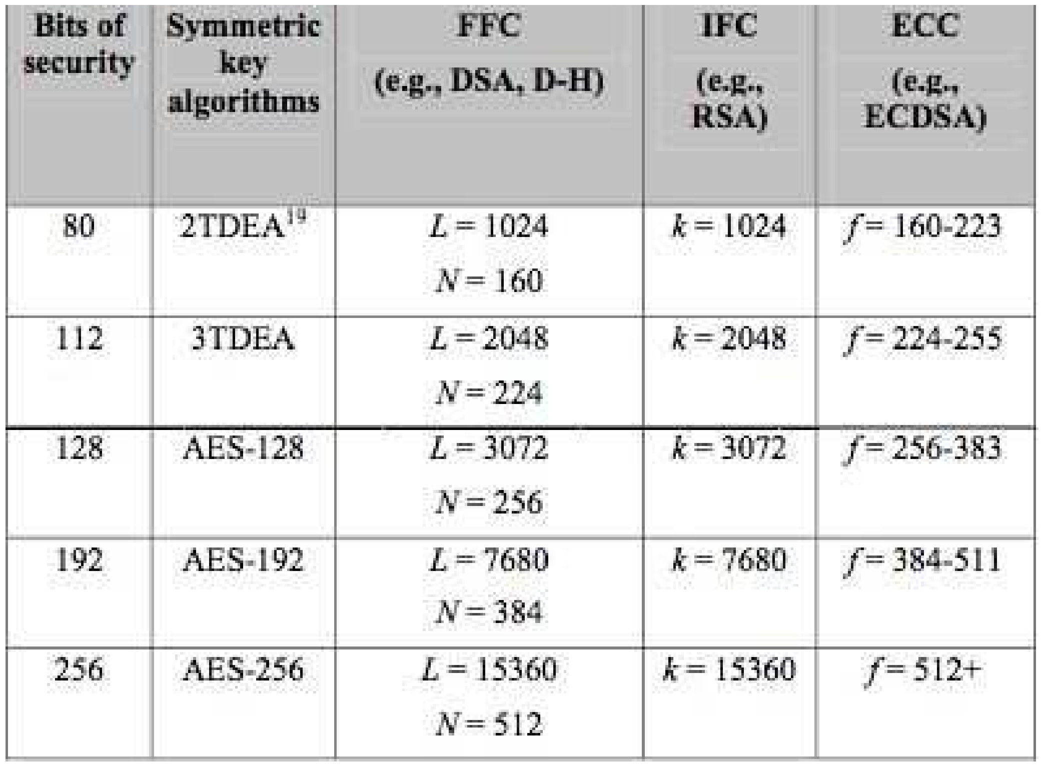


Table 2-1 Recommended Key Sizes

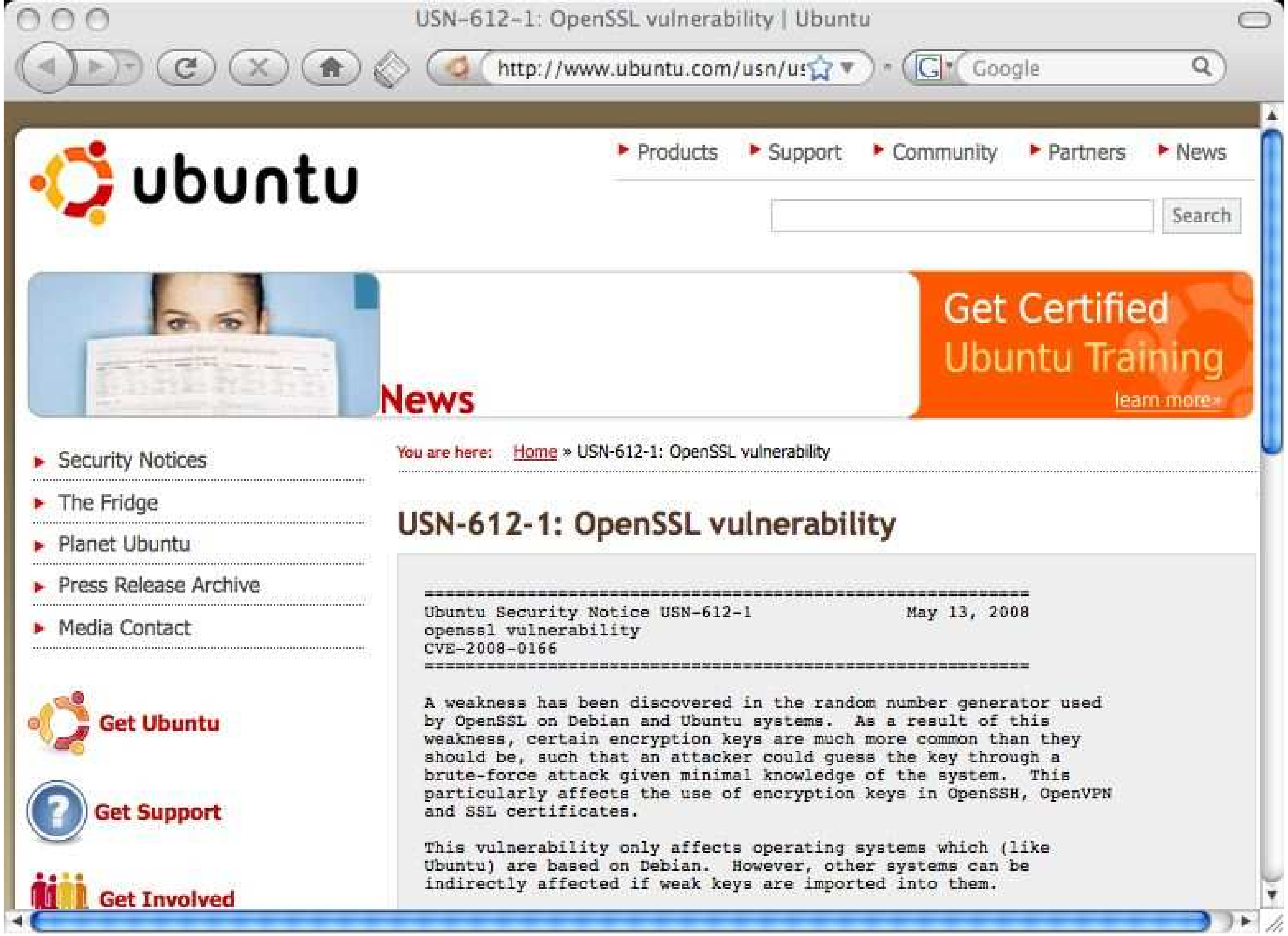
Key Type	Time Period for Use	Algorithms and Key Sizes
Authentication keys (for Users or Devices)	Through 12/31/2013	RSA (1024 or 2048 bits) ECDSA (Curve P-256)
	After 12/31/2013	RSA (2048 bits) ECDSA (Curve P-256)
Digital Signature and Key Establishment keys (for Users or Devices)	Through 12/31/2008	RSA signature or key transport (1024 or 2048 bits) Diffie-Hellman (1024 or 2048 bits) ECDSA, ECDH (Curves P-256 or P-384)
	After 12/31/2008	RSA signature or key transport (2048 bits) Diffie-Hellman (2048 bits) ECDSA, ECDH (Curves P-256 or P-384)
CA and OCSP Responder Signing Keys	N/A	RSA: 2048, 3072, or 4096 bits ECDSA: Curves P-256 or P-384

Comparable Key Strengths

From NIST Special Publication 800-57, 2007, *Recommendation for Key Management - Part 1: General (Revised)*.



Bits of security	Symmetric key algorithms	FFC (e.g., DSA, D-H)	IFC (e.g., RSA)	ECC (e.g., ECDSA)
80	2TDEA[19]	$L = 1024$ $N = 160$	$k = 1024$	$f = 160\text{-}223$
112	3TDEA	$L = 2048$ $N = 224$	$k = 2048$	$f = 224\text{-}255$
128	AES-128	$L = 3072$ $N = 256$	$k = 3072$	$f = 256\text{-}383$
192	AES-192	$L = 7680$ $N = 384$	$k = 7680$	$f = 384\text{-}511$
256	AES-256	$L = 15360$ $N = 512$	$k = 15360$	$f = 512+$



USN-612-1: OpenSSL vulnerability

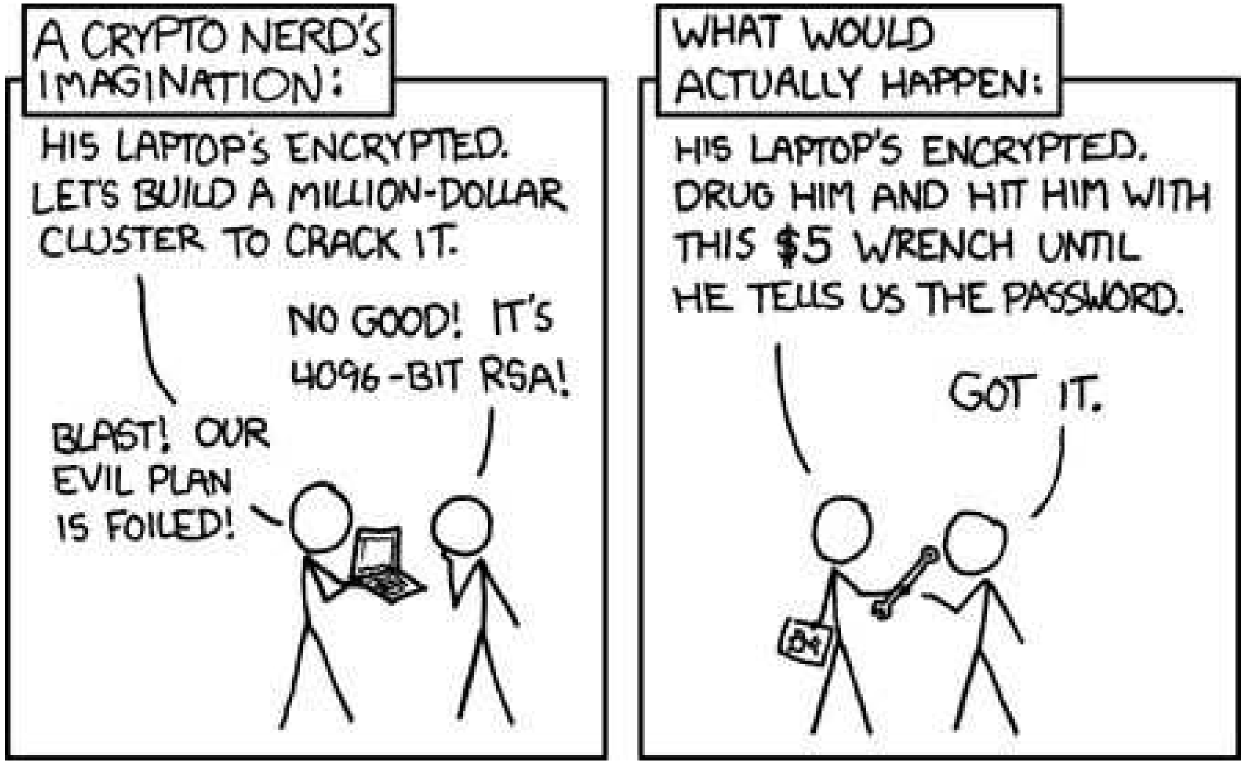
```
===========================================================
Ubuntu Security Notice USN-612-1                May 13, 2008
openssl vulnerability
CVE-2008-0166
===========================================================

A weakness has been discovered in the random number generator used
by OpenSSL on Debian and Ubuntu systems.  As a result of this
weakness, certain encryption keys are much more common than they
should be, such that an attacker could guess the key through a
brute-force attack given minimal knowledge of the system.  This
particularly affects the use of encryption keys in OpenSSH, OpenVPN
and SSL certificates.

This vulnerability only affects operating systems which (like
Ubuntu) are based on Debian.  However, other systems can be
indirectly affected if weak keys are imported into them.
```

Developer accidently removed lines that referenced un-initialized memory that provided entropy for random number generation used to generate primes p and q. This makes it possible to brute force factor public modulus to determine p and q and calculate the private key.

Rubber Hose Cryptography [xkcd.com]


A CRYPTO NERD'S IMAGINATION:
HIS LAPTOP'S ENCRYPTED. LET'S BUILD A MILLION-DOLLAR CLUSTER TO CRACK IT.
NO GOOD! IT'S 4096-BIT RSA!
BLAST! OUR EVIL PLAN IS FOILED!
WHAT WOULD ACTUALLY HAPPEN:
HIS LAPTOP'S ENCRYPTED. DRUG HIM AND HIT HIM WITH THIS $5 WRENCH UNTIL HE TELLS US THE PASSWORD.
GOT IT.

Example: Public Key Cryptography Standards PKCS1/RFC3447

PKCS#1 ... PKCS#15 are a set of standards that specify safe ways of encoding encrypted material. They are designed to avoid known threats: one does not have to be a cryptographer to use RSA safely.

For example: we want to avoid pasword guessing; encrypted material can be vulnerable if it is significantly shorter than public modulus (eg, a secret key). PKCS1 requires that the message include at least 8 bytes of random data.

RFC3447 publishes full details of PKCS1 [`http://www.ietf.org/rfc/rfc3447.txt`]

For simplicity, we let $\{P\}_K$ denote a suitable implementation of the RSA function applied to data P using key K. We denote the inverse of key K as K^{-1} (and the inverse of K^{-1} is K). Thus $P = \{\{P\}_K\}_{K^{-1}}$.

Since the RSA encryption and decryption operations are identical we don't make any special distinction between K and K^{-1} and thus we also have that $D = \{\{D\}_K^{-1}\}_K$

Public Key Attack (Forward Search)

Suppose that Alice and Bob follow the protocol:

$$Alice \rightarrow Bob : \{\{M\}_{K_A^{-1}}\}_{K_B}$$

Eve tests lots of random values R_j until $\{R_j\}_{K_A}$ looks like a valid message.

$$\{R_j\}_{K_A} = \texttt{buy shares}$$

Eve, pretending to be Alice, then sends $M = \{R_j\}_{K_B}$ to Bob.

Bob, thinking that the message originated from Alice computes

$$\{M\}_{K_B^{-1}} = R_j$$

and then to check signature from A

$$\{R_j\}_{K_A} = \texttt{buy shares}$$

Solution: Add redundancy to message

$$Alice \rightarrow Bob : \{\{M, Alice\}_{K_A^{-1}}\}_{K_B}$$

Public Key Cryptography: Encrypting Bulk Data

Public key algorithms involve computationally intensive calculations and, therefore, its not effective to encrypt entire message using public key cipher.

In practice, a symmetric cipher should be used to perform bulk encryption, while the public key cipher is used to secure the symmetric session key. In SW DES about 100 times faster than RSA; difference even greater in HW.

For example, assuming that Bob knows Alice's public key K_A, then, Bob proposes that Alice should use symmetric key K_{AB}:

$$\begin{array}{lll} \text{Msg1}: & B \rightarrow A: & \{K_{AB}, N_A\}_{K_A} \\ \text{Msg2}: & A \rightarrow B: & \{N_A, Msg\}_{K_{AB}} \end{array}$$

Alice uses her private key K_A^{-1} to setup session key K_{AB}.

This is not entirely effective, there is authentication of Alice but Bob is not authenticated: Bob knows that he shares a key with Alice, not vice-versa.

This one-sided authentication is not unlike the secure browsing relationship between a web-browser (Bob) and website (Alice).

Example: A Public Key Protocol with a flaw

There are public-key versions of Needham-Schroeder/Kerberos and we will study SSL later. To help understand the requirements, the following is a sample protocol with a flaw.

Goal is to provide secure and authentic key exchange. Assume everyone knows the public key of everyone else. Use public key encryption to exchange (symmetric) keys; T_A a timestamp for freshness.

$$\text{Msg } 1 \quad A \to B: \quad \{\{T_A, K_{AB}, A\}_{K_A^{-1}}\}_{K_B}$$

In receiving $\{T_A, K_{AB}, A\}_{K_A^{-1}}$, Bob believes that the key K_{AB} could only have been proposed by/come from Alice.

By encrypting K_{AB} with Bob's public key in $\{\{T_A, K_{AB}, A\}_{K_A^{-1}}\}_{K_B}$, Alice believes that the only person able to discover K_{AB} will be Bob.

Therefore, if Alice subsequently receives a message encrypted under symmetric key K_{AB} then it seems reasonable for her to believe it came from Bob, and vice-versa. However, ...

Example: A Public Key Protocol with a flaw

Consider the protocol:

$$\text{Msg } 1 \quad A \to B: \quad \{\{T_A, K_{AB}, A\}_{K_A^{-1}}\}_{K_B}$$

At some point in the past Alice contacts Eve:

$$\text{Msg } \alpha 1 \quad A \to E: \quad \{\{T_A, K_{AE}, A\}_{K_A^{-1}}\}_{K_E}$$

Eve decrypts message to give $X = \{T_A, K_{AE}, A\}_{K_A^{-1}}$.
Eve encrypts X with K_B and sends

$$\text{Msg } \gamma 1 \quad A[E] \to B: \quad \{\{T_A, K_{AE}, A\}_{K_A^{-1}}\}_{K_B}$$

Eve can now masquerade as Alice to Bob! Fix it!

RSA Digital Signatures

Principal A (customer) generates RSA key pair K_A^{-1}, K_A.

A signs a message M (eg, an order) $RSA(K_A^{-1}, M)$.

In practice, A signs a digest, eg, $RSA(K_A^{-1} SHA1(M))$.

Send signed message to Principal B (Merchant)

$$A \rightarrow B : M, \boxed{RSA(K_A^{-1}, SHA1(M))}$$

Principal B uses A's public key to confirm signature.

$$SHA1(M) = RSA(K_A, \boxed{RSA(K_A^{-1}, SHA1(M))})$$

Other public-key schemes can be used to provide digital signatures, for example, the Digital Signature Standard is based on a specialised public key cipher designed specifically for signatures.

We often use the notation $\{M\}_{sK_A}$ to denote the message M signed by the owner of public key K_A.

Recommended Key Sizes

From NIST Special Publication 800-57, 2008, *Recommendation for Key Management - Part 3: Application-specific key management guide (draft)*.

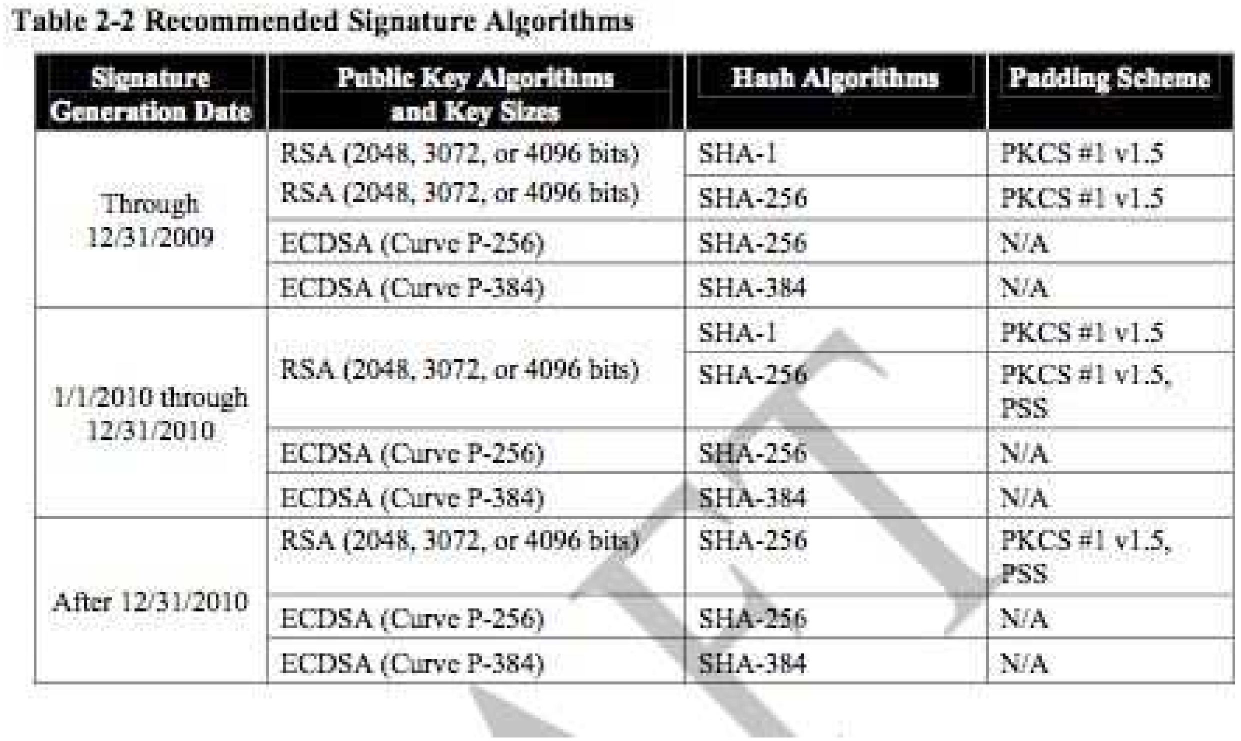


Table 2-2 Recommended Signature Algorithms

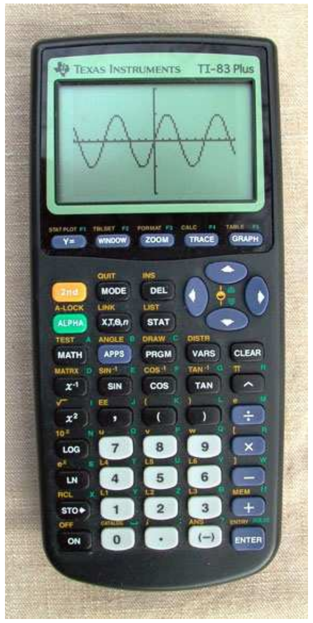
Signature Generation Date	Public Key Algorithms and Key Sizes	Hash Algorithms	Padding Scheme
Through 12/31/2009	RSA (2048, 3072, or 4096 bits)	SHA-1	PKCS #1 v1.5
	RSA (2048, 3072, or 4096 bits)	SHA-256	PKCS #1 v1.5
	ECDSA (Curve P-256)	SHA-256	N/A
	ECDSA (Curve P-384)	SHA-384	N/A
1/1/2010 through 12/31/2010	RSA (2048, 3072, or 4096 bits)	SHA-1	PKCS #1 v1.5
		SHA-256	PKCS #1 v1.5, PSS
	ECDSA (Curve P-256)	SHA-256	N/A
	ECDSA (Curve P-384)	SHA-384	N/A
After 12/31/2010	RSA (2048, 3072, or 4096 bits)	SHA-256	PKCS #1 v1.5, PSS
	ECDSA (Curve P-256)	SHA-256	N/A
	ECDSA (Curve P-384)	SHA-384	N/A

512 bit TI signing key factored [2009]

Updates for Texas Instruments programmable calculators must be signed by a (512 bit) private RSA key known only by TI. The calculator stores a (tamper-proof) copy of TI's public key and uses it to check the signature on updates. If not signed correctly then the calculator will not accept the update.

In 2009 an enthusiast factored the modulus in the public key and used the results to compute the private key. This was done by a brute-force search, taking several months on a fairly standard desktop PC using off the shelf factoring software.

The key was published on the enthusiast's website. TI's response was to have it lawyers send a DMCA take-down to the enthusiast, resulting in the Streisand effect (key subsequently published in a number of other places).



Who Generates the Public/Private Keys?

We call the principal who knows the private key K_A^{-1} (and the public key) the *owner* of the public key K_A.

In principle, the intended owner of public key K_A should be the principal that generates the public/private key pair (K_A^{-1}, K_A).

- ☐ If a third party generated the key pair on behalf of A, then principal A would have to trust the third party not to reveal the private key to anyone else.
- ☐ We would like to be able to believe that when we confirm that $\{M\}_{sK_A}$ was signed by the owner of K_A then the principal cannot easily repudiate their signature. This is subject to them not having declared their private key to be compromised.

If no ambiguity can arise then $\{M\}_{sK_A}$ is used to denote the signed message (by the owner of private key K_A). For example, $M, RSA(K_A^{-1}, SHA1(M))$.

Digital Signatures versus MAC

Customer and Merchant share secret key K for keyed hash function $h_K(M)$

$$\text{Customer} \rightarrow \text{Merchant} : order, h_K(order)$$

Customer and merchant can detect modifications by attacker.

MAC does not constitute evidence that a third party could use to decide whether customer or merchant generated message.

How can customer prove to judge that she ordered only 5 widgets when the merchant claims that it was 10, and both have MACs to (supposedly) prove it? MAC provides integrity check, not authentication check.

Digital Signature Scheme: A third party can resolve disputes about the validity of a digital signature without having to know the signer's key. Digital signature schemes should ideally support non-repudiation.

Be Careful What you Sign

Suppose that Alice and Bob know each others public key and want to exchange signed messages in secret. Need a combination of public key cryptography (for signing) and symmetric key cryptography (for secrey, etc).

Alice generates a symmetric session key K_{AB} which is used to encrypt the bulk data. Two strategies:

$$A \to B : \{M\}_{K_{AB}}, \{\{h(M), K_{AB}, A, B, \ldots\}_{K_A^{-1}}\}_{K_B} \quad (1)$$

$$A \to B : \{M, h(M)\}_{K_{AB}}, \{\{K_{AB}, A, B, \ldots\}_{K_A^{-1}}\}_{K_B} \quad (2)$$

Which protocol provides an effective signature?

Some Public Key Protocols

ISO/IEC 9798-3 Signature Key Three Pass MA Protocol. Alice and Bob know each other's public keys K_A and K_B, respectively and run the following protocol to check each other's presence.

Msg1	$A \rightarrow B$	A, N_a
Msg2	$B \rightarrow A$	$\{A, N_a, N_b\}_{sKb}$
Msg3	$A \rightarrow B$	$\{N_a, N_b\}_{sKa}$

This protocol supports optional attributes in the messages and can be used to, for example, carry out an authentic DH key exchange.

Msg1	$A \rightarrow B$	A, N_a
Msg2	$B \rightarrow A$	$\{A, N_a, N_b, g^y \bmod n\}_{sKb}$
Msg3	$A \rightarrow B$	$\{N_a, N_b, g^x \bmod n\}_{sKa}$

where g and n give DH generator and modulus, respectively. x and y are the secret exponents of A and B. On completion, A holds authenticated $g^y \bmod n$ from B, and similarly for B; both share secret $g^{xy} \bmod n$.

Some Public Key Protocols: A Preliminary Sketch of SSL

Alice uses an SSL-style protocol to connect to the web-server Bob.

Msg1	$A \rightarrow B$	$A, \{A, N_A, K_{AB}, \ldots\}_{K_B}$
Msg2	$B \rightarrow A$	$\{B, N_A, N_B \ldots\}_{K_{AB}}$
Msg3	$A \rightarrow B$	$\{A, N_B + 1\}_{K_{AB}}$
...	$A \leftrightarrow B$	$\{data, etc\}_{K_{AB}}$

In this protocol, we assume that Alice knows the Server Bob's public key K_B, which is used to establish a secret shared session key K_{AB}.

This protocol provides authentication of Bob. Alice knows for sure that when she encrypts her messages with K_{AB} then only Bob can read them, since only Bob can decrypt message 1 to discover K_{AB}.

The protocol does not provide authentication of Alice. Bob does not know the initiator of the protocol. If (weak) authentication of the client is required then, once connected, Alice could provide a password/pin at a login web-page. Note that in general SSL can also provide client-side authentication as part of the protocol (we'll see this later).

Station to Station Protocol [Diffie VanOorschot Wiener]

A Diffie-Hellman key exchange that includes an exchange of authentication signatures.

Msg1	$A \rightarrow B$	$g^x \bmod p$
Msg2	$B \rightarrow A$	$g^y \bmod p, \{\{g^x \bmod p, g^y \bmod p\}_{sK_B}\}_K$
Msg3	$A \rightarrow B$	$\{\{g^x \bmod p, g^y \bmod p\}_{sK_A}\}_K$

where Alice and Bob own public keys K_A and K_B, respectively, and are somehow known to each other.

The generator g and modulus p are publicly known/exchanged. Alice and Bob generate local secrets x and y, respectively, and the shared DH secret is $K = g^{xy} \bmod p$.

Features of Station to Station Protocol

Perfect Forward Secrecy Disclosure of long-term secret keying material does not compromise secrecy of exchanged keys from earlier runs[1].

Direct Authentication Authentication is established by the end of the protocol run. Both parties have proven that they know the secret key.

No Timestamps

[1] Unlike, the protocol $A \rightarrow B : \{\{A, B, K\}_{K_A^{-1}}\}_{K_B}$ where an attacker that discovers K_B^{-1} can extract earlier keys from earlier encrypted exchanges.

Station to Station Protocol in Practice

Msg1	$A \rightarrow B$	$g, p, , g^x \bmod p$
Msg2	$B \rightarrow A$	$Cert_{K_B} g^y \bmod p, \{\{g^x \bmod p, g^y \bmod p\}_{sK_B}\}_K$
Msg3	$B \rightarrow A$	$Cert_{K_A}, \{\{g^x \bmod p, g^y \bmod p\}_{sK_A}\}_K$

where $Cert_{K_A} = \{Alice, K_A, g, p\}_{sK_T}$, where K_T is a trusted third party. and g, p must be signed.

Can run an authentication-only version of STS,

Msg1	$A \rightarrow B$	R_A
Msg2	$B \rightarrow A$	$Cert_{K_B}, R_B, \{R_B, R_A\}_{sK_B}$
Msg3	$A \rightarrow B$	$Cert_{K_A}, \{R_A, R_B\}_{sK_A}$

which is essentially ISO/IEC 9798 Three Message MA protocol (which includes optional fields for symmetric key to be exchanged).